

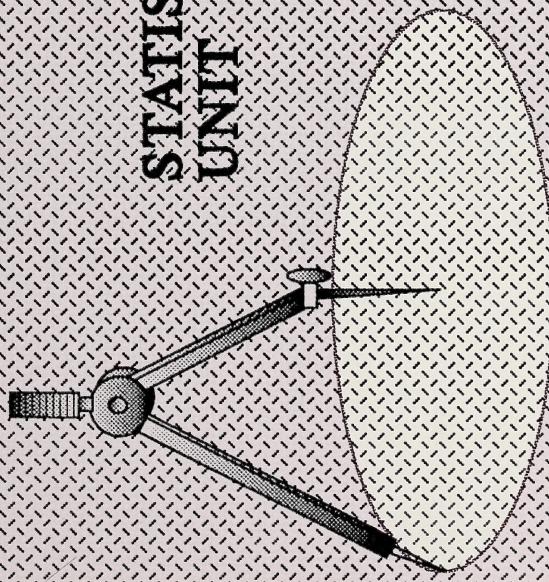
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MATHEMATICS 03

STATISTICS UNIT 8



Alberta
EDUCATION



Welcome



You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 23 Student Module Unit 8 Statistics Alberta Distance Learning Centre ISBN No. 0-7741-0012-5 *1992

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know and Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each Topic, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to Activities (**Appendix A**) and any other charts, tables, etc. which may be referred to in the topics (**Appendix B**, etc.).

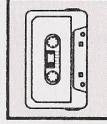
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

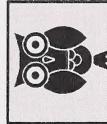
An explanation of what they mean is written beside each visual cue.



Key Idea
• flagging important ideas



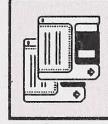
Audiotape
• learning by listening to an audiotape



What You Already Know
• reviewing what you already know



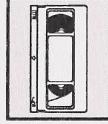
Another View
• exploring different perspectives



Computer Software
• learning by using computer software



Videotape
• learning by viewing a videotape



Print Pathway
• choosing a print alternative



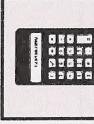
Calculator
• using your calculator



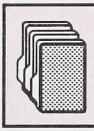
Exploring the Topic
• actively learning new concepts



Extra Help
• providing additional study



What Lies Ahead
• previewing the unit



Extensions
• going on with the topic



What You Have Learned
• summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14% <hr/> $\overline{100\%}$

Unit Assessment

After completing this unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

- Unit assignment - 50%
- Supervised unit test - 50%

Introduction to Statistics

This unit covers topics dealing with statistics. Each topic contains explanations, examples, and activities to assist you in understanding statistics. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 8 Statistics

Contents at a Glance

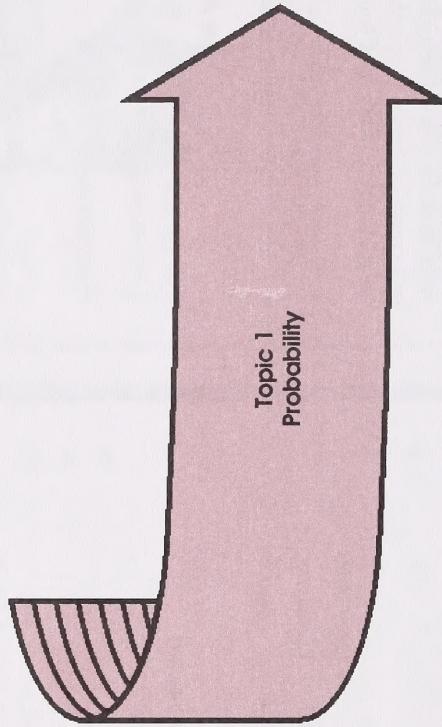
Value	Statistics	3
	What You Already Know	5
	Review	5
100%	Topic 1: Probability	7
	• Introduction	• Extra Help
	• What Lies Ahead	• Extensions
	• Exploring Topic 1	
	Unit Summary	37
	• What You Have Learned	
	• Unit Assignment	
	Appendices	38
	• Appendix A	39
	• Appendix B	51

Statistics

A picture speaks louder than words. In the last course you read and created graphs. The graphs spoke. You interpreted their voice to make some conclusion about the data. In this unit you will create your own data and make some conclusions or predictions about the data you collect.



Unit 8 Statistics



What You Already Know



Refresh your memory!

Do remember how to do the following?

1. Convert fractions to decimals or percent.
2. Solve proportion questions which involve percent.
3. Use the vocabulary of probability as it applies to the likelihood of the occurrence of an event within a population.

Now that you have looked at material you studied previously, do the Review to confirm your understanding of this material.

Try the following review questions.

1. Change the following fractions to decimals.

a. $\frac{2}{5}$

b. $\frac{3}{4}$

c. $\frac{2}{3}$

d. $\frac{17}{23}$

e. $\frac{287}{298}$

2. Change the following fractions to percents.

a. $\frac{34}{100}$

b. $\frac{12}{17}$

c. $\frac{9}{20}$

d. $\frac{19}{27}$

e. $\frac{123}{234}$

3. J. B. Card High School has a population of 1250. There are 720 girls. What percentage of the school population is girls?

4. 38% of the people surveyed selected Crost dentifrice. 2000 people were surveyed. How many people selected Crost dentifrice?

5. Three out of eight students drive to school. How many students from a group of 13 000 would drive?

Review



6. If two out of eleven people prefer red cars, four out of eleven prefer black cars, and five out of eleven prefer blue cars, what colour of car should a lottery select?

If you do not understand how to do or use any of the above questions, please return to the Data Management Unit of the Mathematics 9 course for a more extensive review before you start this unit.

Now turn to the **Review Solutions** in **Appendix A**.



Topic 1 Probability

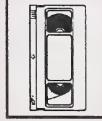
Introduction



Jeanne Dixon predicts that those people born under the sign of Capricorn will be successful! Your teacher predicts that you will be successful in mathematics if you do your homework!

Predictions are made everyday. You too can make predictions after you complete your study of probability.

If you have access to a video cassette player you may wish to view the video titled *Of Dice and Men*¹ for a general overview of probability.



What Lies Ahead



Throughout the topic you will learn to

1. determine the probability of an event that is a number between 0 and 1 which describes the likelihood of the occurrence of that event
2. find the probability of two or more events occurring together by the application of the multiplication law for independent and dependent events
3. find the probability of the occurrence of one or the other of two events by the application of the addition law
4. design and carry out simulations involving events which have known and unknown probabilities

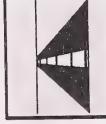
Now that you know what to expect, turn to the next page to begin your study of probability.

¹ *Of Dice and Men* is a title of the National Film Board.

Exploring Topic 1



Activity 1



Determine the probability of an event that is a number between 0 and 1 which describes the likelihood of the occurrence of that event.

Roll a six-sided die 50 times and record in the following chart the number of times each side faces up.

Number of dots per face	1	2	3	4	5	6
Number of times face was up						

Use the data that you have just collected to answer the following questions.

1. Did you get all 6 numbers? Did you expect all six numbers to be rolled?
 - I will probably pass this course.
 - It will likely rain today.
 - I may possibly go to the movies on Saturday.
 - There is a 50-50 chance that our team will win the volleyball match today.
2. Did one number come up more frequently than the others?
 - Would you expect one number to come up more often than the rest?
3. Calculate the fraction of rolls which were 1's, 2's, 3's, 4's, 5's, and 6's. For example, find the fraction for the 1's.
$$\text{Fraction of 1's} = \frac{\text{total number of 1's}}{\text{total number of rolls}}$$
4. What do you think would happen if you conducted this experiment again? Would you expect these fractions to be the same? Try it and see.

Take a look at probability through an experiment. Your experiment will be tossing of a fair die. You say fair because there is an equal chance of the die falling on each of the six sides.



You should get results similar to the following:

1. All of the numbers should come up.
2. The six faces did not appear an equal number of times.
3. Since the six faces did not appear an equal number of times, the fractions were different, but all were around one sixth.
4. If you repeated this experiment a number of times, you would not get the exact same results.

Take a closer look at the results of this experiment. The fractions obtained in question 3 of the experiment are an estimate of the chance or probability of obtaining a given number.

Suppose that you rolled the face with five dots 9 times. The fraction for the 5's would be

$$\frac{9}{50} = 0.18 \text{ or } 18\%.$$

If you rolled the die 1000 times, you would expect the face with five dots to come up 18% of the time or 18% of 1000 times.

$$0.18 \times 1000 = 180$$

You would expect the face with five dots to come up 180 times.

The method that you used for finding the fractions for the number of times a particular face came up can be written generally as

$$\text{fraction} = \frac{\text{number of desirable results}}{\text{total number of results}}$$

This is an exact fraction that is used to find **experimental probability**.



experimental probability of an event

$$= \frac{\text{number of desirable results}}{\text{total number of results}}$$

Experimental probability is found when you cannot calculate the exact or theoretical probability of a situation. The experimental probability is found by doing exactly what you did here. You perform an experiment or a number of experiments to find the number of times the desirable results appear out of the total number of results.

For this particular situation, you can find the **theoretical probability**. The theoretical probability is calculated in the same way as shown below.



theoretical probability of an event

$$= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Theoretical probability differs in the following ways.

1. You do not perform an experiment to find the results.
2. The different outcomes must be equally likely (like the die).

See what the theoretical probability is of rolling five dots on the face of a die. There is a total number of six faces on the die which are equally likely to be rolled. There is only one face which has five dots. Therefore, the probability of rolling a 5 = $\frac{1}{6}$ and is written $P(5) = \frac{1}{6}$.

Why are these two probabilities different?



The theoretical probability will always be the same since you are looking at the possible outcomes to the situation.

For the experimental probability, you will come up with different answers for different experiments. The last experiment you had a probability of 0.18 for the five being rolled. In the next experiment it might become 0.22 and in the third experiment it might become 0.14. Every time you conduct an experiment, you should expect to get a different probability.

Why would you want to use experimental probability?

There are circumstances where you cannot calculate theoretical probability. Look at the case of calculating the probability of electing a particular person to become prime minister. The outcomes are not equally likely and means you cannot use theoretical probability. But, you can use experimental probability. You can survey a part of the population and find out who they're going to vote for. Then you can change those numbers into fractions that would represent the probability of each person being elected prime minister.

Why are the numbers different for experimental and theoretical probability?

It's not a case that the fractions are different; it's a case that you did not roll the die enough times in your experiment. The more times you roll the die in the experiment, the closer the experimental probability fraction will get to the theoretical probability fraction.

In your experiment, you rolled the die 50 times and obtained an experimental probability of 0.18 for rolling a five. If you rolled the die 500 times you might get an experimental probability of 0.17 and if you rolled the die 50 000 times you might get an experimental probability of 0.165. As the number of rolls increase in the experiment, you get closer and closer to the value of the theoretical probability of $0.1\overline{6}$.

The theoretical probability would be the number you get if the die was rolled an infinite number of times in the experiment. You have been introduced to some number terms here; define them before moving onto something new.

In this situation, the sample space was the numbers 1, 2, 3, 4, 5, and 6 – the number of dots on the faces of the die. The sample space is all of the possible outcomes of the experiment or situation.

The outcomes are the different results that can be obtained. All of the different outcomes will be listed in the sample space.

The event is all of the favourable outcomes. In the case that you just looked at, rolling a five was the event.

The favourable outcomes are all of the results that you want to appear. The favourable outcomes would be the side of the die with five dots. You now have enough information to define probability.

Probability is a branch of mathematics concerned with the study of the chance that a given event will occur.

From the fraction that you found for probability, probability can be defined as:

What is the probability of rolling a 1, 2, 3, 4, 5, or 6 on the die?

The **probability** of an event happening is the total number of favourable outcomes divided by the total number of outcomes where all of the outcomes have an equal chance of happening.

So far, with the experiment, you got a proper fraction that was less than one. What values can the fraction take?

Do some calculations to find out.

What is the probability of rolling a face that has seven dots?

There are no faces that have seven dots; therefore,

$$\begin{aligned} P(7) &= \frac{0}{50} \\ &= 0. \end{aligned}$$

This number represents the probability when the event is not present in the sample space. Since there is no way you can get the event out of the sample space, it is called an **impossible event** and is the smallest probability fraction possible.



The word **or** means that any one of the outcomes is acceptable.
The word **and** means that all of the outcomes must be present.

Since there is no way that you can roll the die and not get a number that is part of the event, this is called a **certain event** and is represented by the number 1. This is the largest number that you can get with probability.

All numbers that you use to represent a probability are going to be between the numbers 0 and 1.



$$\begin{aligned} P(1, 2, 3, 4, 5 \text{ or } 6) &= \frac{6}{6} \\ &= 1 \end{aligned}$$

Now try some questions.

Do any 4 of the following questions.

- a. Put the results into the following chart.

Any tables you may need are provided in **Appendix B**.

1. Using a six-sided die, what is the probability of the following:

- a. rolling a 4
- b. rolling a 2

2. Using a coin, what is the probability of the following:

- a. tossing a head
- b. tossing a tail

- i. tossing a head
- ii. tossing a tail

3. Using a deck of 52 cards, what is the probability of the following:

- a. selecting the ace of spades
- b. selecting an ace
- c. selecting a spade
- d. selecting a black card

4. Toss a coin 100 times.

- a. Put the results into the following chart.

Face of Coin	Heads	Tails
Number of Times Facing Up		

A deck of 52 cards contains the following:

- 13 clubs, 13 diamonds, 13 hearts, and 13 spades
- 26 black (clubs and spades) cards, and 26 red (diamonds and hearts) cards
- Each suit has one ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king.

5. Define in your own words the phrases **experimental probability** and **theoretical probability**.



For solutions to Activity 1, turn to **Appendix, Topic 1**.

Activity 2



Find the probability of two or more events occurring together by the application of the multiplication law for independent and dependent events.

Sometimes two or more simple events are combined and treated as a single event. Such an event is called a **compound event**. Here are some examples of compound events:

- the tossing of two coins
- the drawing of a number from a hat
- the rolling of a die

The next experiment involves a compound event.

Consider rolling two dice – one red, and one white. These may be treated as a single event even though each die contributes an outcome. Look at all the possible outcomes for a two dice experiment.

On the white die, there are six possible outcomes – 1, 2, 3, 4, 5, and 6. On the red die, there are the same six possible outcomes.

What happens when you combine these two dice? How many outcomes are possible?

The next two diagrams (the tree diagram and the chart) display the outcomes in different pictorial forms.

In the tree diagram, the first row of numbers will represent the possible outcomes that the red die can have. The second row will represent all of the possible outcomes that the white die can have, after the red die.

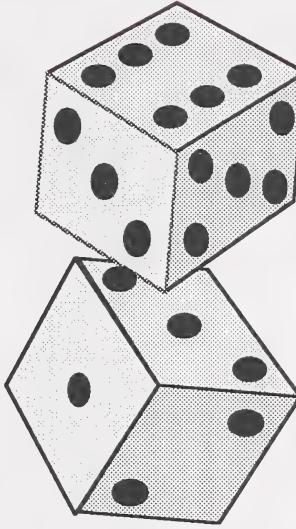
The tree diagram is on the following page.

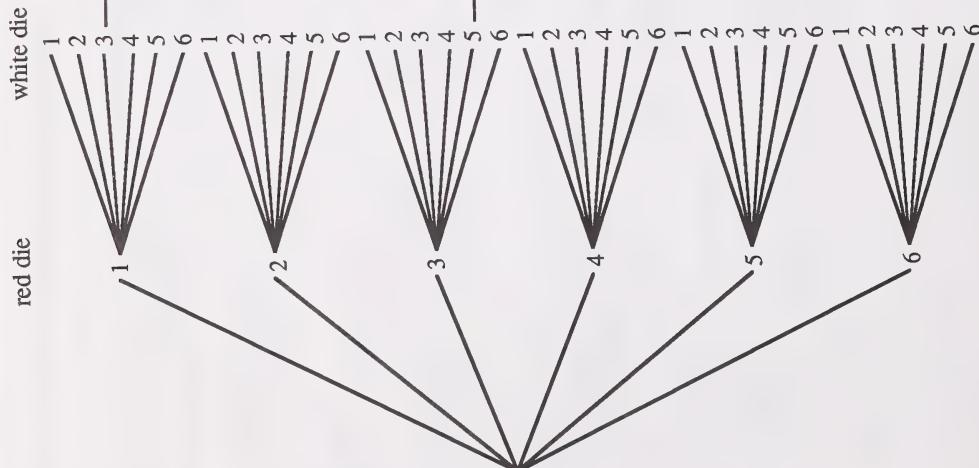
To find any outcome on the tree diagram:

1. Start on the left side of the diagram and follow one of the limbs of the tree to one of the outcomes of the red die.
2. Next follow one of the limbs to one of the outcomes on the white die.



The combination of the two numbers on the two dice give you that particular outcome.





This position on the tree diagram represents the outcome where you get a 1 on the red die and a 3 on the white die.

This position on the tree diagram represents the outcome where you get a 3 on the red die and a 5 on the white die.

If you count all of the numbers under the white die column, you will get the total number of possible outcomes, 36.

Since there is only one route you can follow to get a red three and a white 5, the probability is $\frac{1}{36}$.

However, if you are looking for the probability of getting a white 6, then you can go down all six of the limbs to the red die, but you must select one particular route from each red die to the number 6 on the white die. In this case, you have managed to get to every six in the white die column, a total of six different routes. Therefore,

$$\begin{aligned} P(\text{white } 6) &= \frac{6}{36} \\ &= \frac{1}{6}. \end{aligned}$$

This is the same probability as getting the number six on one die. This happens because you do not care what number comes up on the red die.

The chart consists of ordered pairs. The first coordinate of the ordered pair represents the red die and the second coordinate represents the white die.

(Red, White)	(, White)
(, 1)	(, 2)
(1,)	(1, 2)
(1, 1)	(1, 3)
(2,)	(2, 3)
(2, 1)	(2, 4)
(3,)	(3, 3)
(3, 1)	(3, 4)
(4,)	(4, 3)
(4, 1)	(4, 4)
(5,)	(5, 2)
(5, 1)	(5, 3)
(6,)	(6, 3)
(6, 1)	(6, 4)
(6, 2)	(6, 5)
(6, 3)	(6, 6)

The ordered pair (5, 2) represents a 5 on the red die and a 2 on the white die.

If you count all of the possible outcomes you will again come up with the number 36 (count the number of ordered pairs in the lower right box). Therefore,

$$P(5, 2) = \frac{1}{36}.$$

What is the probability of getting a 4 on the red die?

In this case you look for all of the ordered pairs that have a 4 on the first coordinate. There are 6 of them and they are all in the same row. Therefore,

$$\begin{aligned} P(4, \cdot) &= \frac{6}{36} \\ &= \frac{1}{6}. \end{aligned}$$

Is there another way of finding probabilities without having to make a tree diagram or a chart?

Examine the different parts of the two probabilities that you have just found.

You already know that

$$P(5, 2) = \frac{1}{36}.$$

Find the probability of getting a 5 on the red die.

$$P(\text{red } 5) = \frac{1}{6}$$

Now the probability of getting a 2 on the white die is

$$P(\text{white } 2) = \frac{1}{6}.$$

How can you find $P(5, 2)$ using $P(\text{red } 5)$, and $P(\text{white } 2)$?

$\frac{1}{36}$ is the product of $\frac{1}{6}$ and $\frac{1}{6}$.

You can also write

$$P(5,2) = P(\text{red } 5) \times P(\text{white } 2).$$

Or you can write it more generally

$$P(A,B) = P(A) \times P(B).$$

See if this holds true for other cases.

$$P(4, \text{any number}) = P(\text{red } 4) \times P(1,2,3,4,5 \text{ or } 6)$$

$$= \frac{1}{6} \times 1$$

$$= \frac{1}{6}$$

The equation also worked in this case. In fact, this equation will work in all cases where the two are independent events.

Independent events are events that do not have any influence over the outcome of the other event.



The Multiplication Law
 $P(A \text{ and } B) = P(A) \times P(B)$, where A and B are independent events.

You should not be concerned about what number appears on a single die, but instead the sum of the two numbers. The following chart has the sum of the two dice instead of the ordered pairs.

Sum of Red and White dice	White					
	1	2	3	4	5	6
Red	1	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11

Notice that there are still 36 entries in the lower right box, but now there is no way to distinguish between which location a particular sum will fit.

What is the probability of getting a sum of seven on the two dice? There are six locations on the chart where there is a sum of seven. Therefore,

$$P(7) = \frac{6}{36}$$

$$= \frac{1}{6}.$$

What is the probability of getting a sum of 11? In this case, there are only 2 locations in the chart. Therefore,

$$P(11) = \frac{2}{36}$$

$$= \frac{1}{18}.$$

Examine the case where a box contains 19 marbles. There are 4 red, 7 green, and 8 yellow marbles.

What is the probability of drawing a green marble after a yellow marble has been drawn?

Find the probability of drawing a yellow marble.

$$P(\text{yellow}) = \frac{8}{19}$$

Now there are only 18 marbles in the box. Therefore,

$$P(\text{green} / \text{yellow}) = \frac{7}{18}.$$

The expression $P(\text{green} / \text{yellow})$ is read as the probability of drawing a green marble after a yellow marble has been drawn.

These two probabilities must be multiplied to find out what the probability is of both of them happening in sequence.

$$P(\text{yellow and green}) = P(\text{yellow}) \times P(\text{green} / \text{yellow})$$

$$= \frac{8}{19} \times \frac{7}{18}$$

$$= \frac{56}{342}$$

This case demonstrates another aspect of the multiplication law. This type of equation is used only when one event is **dependent** on the other.

Dependent events are events that have influence over the outcome of the other event.

This formula can be written as follows.



The Multiplication Law

$$P(A \text{ and } B) = P(A) \times P(B/A), \text{ where A and B are dependent events.}$$

The expression $P(B/A)$ is read as the probability of event B happening after event A has happened.



This type of probability is called a **conditional probability**, since the outcome of one of the events depends on the outcome of the other event.

What is the probability of drawing two green marbles?

$$\begin{aligned} P(\text{green}_1 \text{ and green}_2) &= P(\text{green}_1) \times P(\text{green}_2 / \text{green}_1) \\ &= \frac{7}{19} \times \frac{6}{18} \\ &= \frac{7}{57} \end{aligned}$$

Now try some questions.

Answer any 5 of the following questions.

Any tables you may need are provided in **Appendix B**.

1. Explain the difference between conditional and compound probability.
2. a. Draw a tree diagram for a coin toss.
b. Use the diagram to find the probability of each of the following:
 - i. tossing 3 heads in a row
 - ii. tossing 2 heads and a tail
- c. Throw three coins 50 times and compare the experimental probabilities to the theoretical probabilities that you have just calculated.

3. Use a blue die and a green die to find the probability of the following:

- a. getting a 4 on the blue die and 5 on the green die
- b. getting a 2 on the blue die and an even number on the green die
- c. getting a 1, 5, or 6 on the blue die and a prime number on the green die (Do not include 1 as a prime number.)

4. a. Using two dice, what is the probability of getting the following:

- i. a sum of 6
- ii. a sum of 8
- iii. a sum of 3, 5, or 11

b. Toss two dice 50 times and compare the experimental probabilities to the theoretical probabilities that you have just calculated.

5. a. Using a box that contains 5 yellow, 6 blue, 7 green, and 9 red marbles, what is the probability of each of the following:

- i. drawing a blue marble
- ii. drawing a yellow and blue marble
- iii. drawing 2 red marbles

b. Put the above marbles into a box and select two marbles at a time and then replace them. Do this 50 times and compare the experimental probabilities to the theoretical probabilities that you have just calculated.

6. What is the probability of winning a lottery jackpot. (To win you must select the same six numbers that are drawn. The numbers run from 1 to 49.)

For solutions to Activity 2, turn to Appendix A,
Topic 1.



Activity 3



Find the probability of the occurrence of one or the other of two events by the application of the addition law.

Examine a case involving a deck of cards. Normally, the deck of cards you purchase has 54 cards. The chart on the next page shows the different cards in the deck.

i. For your calculations you will discard the jokers. (The jokers will only be used when it is specifically stated.) This means you will be working with 52 cards.

What is the probability of drawing an ace? Examining the chart you will see that there are four aces, one for each suit (the suits are diamonds, hearts, clubs, and spades).

Therefore,

$$\begin{aligned} P(\text{ace}) &= \frac{4}{52} \\ &= \frac{1}{13}. \end{aligned}$$

What is the probability of drawing an ace or a queen?

$$\begin{aligned} P(\text{ace or queen}) &= \frac{8}{52} \\ &= \frac{2}{13} \end{aligned}$$

The word **or** means that either of the two events are acceptable.
The word **and** means that both events must be present.

You are not always going to have a chart to look at to help you find the number of favourable outcomes. See if you can find another way of getting this answer.

Red Cards				Black Cards			
Diamonds	Hearts	Clubs	Spades	Diamonds	Hearts	Clubs	Spades
ace	2	ace	2	ace	2	ace	2
3	4	3	4	3	4	3	4
5	6	5	6	5	6	5	6
7	8	7	8	7	8	7	8
9	10	9	10	9	10	9	10
jack	queen	jack	queen	jack	queen	jack	queen
king		king		king		king	
				king		joker	joker

$$P(\text{ace}) = \frac{1}{13} \text{ and } P(\text{queen}) = \frac{1}{13}$$

Can you see how these two probabilities can be put together to give you the same solution.

$$\frac{2}{13} = \frac{1}{13} + \frac{1}{13}$$

Therefore,

$$P(\text{ace or queen}) = P(\text{ace}) + P(\text{queen}),$$

or written more generally as

$$P(A \text{ or } B) = P(A) + P(B).$$

There is something that needs to be noted here. Both of these events do not have any cards in common. Therefore, these two events are considered to be **mutually exclusive**.

Mutually exclusive events are events that cannot occur simultaneously.

This gives you the following rule.



The Addition Law
 $P(A \text{ or } B) = P(A) + P(B)$, where A and B are mutually exclusive.

How do you handle those cases where the events are not mutually exclusive?

Solve this problem by finding the probability of drawing an ace or a club.

Using the chart, the following occurs.

$$\begin{aligned}P(\text{ace or club}) &= \frac{16}{52} \\&= \frac{4}{13}\end{aligned}$$

Look at the probabilities of getting each of these events individually.

$$P(\text{ace}) = \frac{4}{52} \quad P(\text{club}) = \frac{13}{52}$$

$$P(\text{ace or club})? P(\text{ace}) + P(\text{club})$$

$$\begin{aligned} &? \frac{4}{52} + \frac{13}{52} \\ &? \frac{17}{52} \end{aligned}$$

This is not the same solution. What is the difference?

If you examine this problem a little more carefully, you will notice that the ace of clubs was counted twice in this last calculation.

You need to find the probability of drawing the ace of clubs and subtracting it so that you are only adding it into the equation once.

$$P(\text{ace clubs}) = \frac{1}{52}$$

$$P(\text{ace or club}) = P(\text{ace}) + P(\text{club}) - P(\text{ace and clubs})$$

$$\begin{aligned} &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \end{aligned}$$

Now you have the proper solution and a new equation to use.



The Addition Law
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ where event A and B are not mutually exclusive.

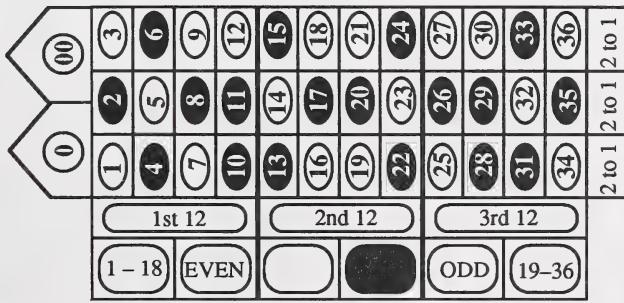
The game of roulette is made of three pieces. A marble, a wheel with slots, and a betting board which is shown at the left.

Roulette is played by spinning the wheel and throwing the marble into the travelling surface around the outer edge of the wheel. As the marble slows down, it will drop into one of 38 slots cut into the lower section of the wheel.

Players place money (or chips) on the betting board in locations that represent where they believe the marble will stop on the wheel.

By looking at the board, you can see that the players have all types of options of how they can place their money. They can place their money on individual numbers, red or black colours, even or odd numbers, individual columns, first and second eighteen numbers and first, second, and third twelve number groups. In addition to this, they can place their money on the lines between these different bets to include both three or four sides of the lines into the bet.

Find the probability of different betting combinations on the board.



Code:

① These are red numbers.

② These are black numbers.

The numbers ① and ⑩ are the only two green numbers on the board. These zeros are not considered even numbers.

What is the probability of the marble landing on the first twelve or a red number?
 $P(\text{first twelve or red}) = P(\text{first twelve}) + P(\text{red}) - P(\text{first twelve and red})$

$$\begin{aligned} &= \frac{12}{38} + \frac{18}{38} - \frac{6}{38} \\ &= \frac{24}{38} \\ &= \frac{12}{19} \end{aligned}$$

The probability of the marble landing on a number in the first twelve or that is red is $\frac{12}{19}$.

What is the probability of the marble landing on the second eighteen or the third column?

$$P(\text{2nd 18 or 3rd col}) = P(\text{2nd 18}) + P(\text{3rd col}) - P(\text{2nd 18 and 3rd col})$$

$$\begin{aligned} &= \frac{18}{38} + \frac{12}{38} - \frac{6}{38} \\ &= \frac{24}{38} \\ &= \frac{12}{19} \end{aligned}$$

The probability of the marble landing on a number in the second eighteen or the third column is $\frac{12}{19}$.

Now try the following questions.

Do any 4 of the questions. Any tables you may need are provided in **Appendix B**.

- Using a deck of cards (52), what is the probability of drawing the following:

- a face card (jack, queen, or king) or a black card
- a number that is a multiple of 3 or a heart
- a red card or a black card
- a card with a number on its face or a king
- a red queen or a diamond

You must actually count the number of squares that are red in the first third of the numbers.

2. Using the roulette wheel, what is the probability of rolling the following:

- a. a number that is a multiple of three or a multiple of 5
- b. a red number or an even number
- c. a number in the second twelve or an odd number
- d. a number in the first column or an even number
- e. a number in the first column or a number greater than 30

3. a. A coin is tossed three times. What is the probability of getting the following:

- i. exactly two or three heads
- ii. exactly two heads or no heads
- iii. exactly two tails or two heads

b. Toss three coins a total of 50 times and see if the experimental probabilities are near the theoretical probabilities.

4. A bag contains 3 white, 4 blue, 5 red, 6 green, and 7 orange marbles.

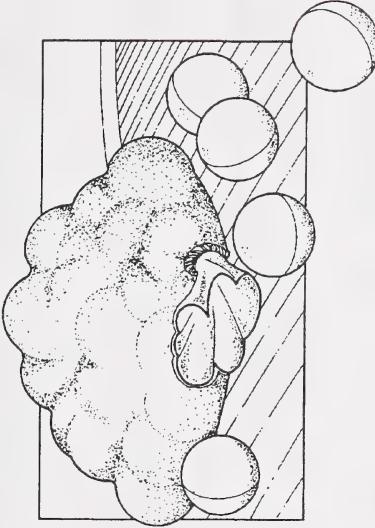
- a. What is the probability of drawing a white or blue marble?
- b. What is the probability of drawing a blue or green marble?
- c. What is the probability of drawing a blue, red, or orange marble?
- d. What is the probability of drawing a white, blue, or orange marble?

5. If you toss a pair of dice, what is the probability that the sum of the dice will be the following:

- a. a 2 or a 5
- b. a prime number or an even number
- c. an odd number or greater than 9
- d. a composite number or less than 2 (The numbers 4, 6, 8, 9, 10, 12 are considered composite.)
- e. a 2, 3, or 11



For solutions to Activity 3, turn to Appendix A,
Topic 1.



Activity 4



Design and carry out simulations involving events which have known and unknown probabilities.

Can you think of another situation where you have two options that are equally likely? The coin toss is one of these situations. The coin toss has exactly two possible outcomes which are equally likely – heads and tails.

Different outcomes will be assigned to both sides of the coin. If the coin lands with the heads facing up, assume that a correct answer has been given on the test. If the coin lands with the tails facing up, assume an incorrect answer has been given on the test.

Sometimes it is impossible to conduct an experiment to collect data.

In these cases, a simulation of the situation is used to collect the data. The following is one such case.

John has decided to guess at the answers on his true-false test. He wants to determine the chances of scoring 60% or better on a 10 question true-false test.

John will not be permitted to write this test more than once and this will not be nearly enough times to permit him to find the necessary probability.

The solution to John's predicament is a simulation. A simulation will permit John to find the necessary probability without actually writing the exam.

Examine this case more carefully and decide on how the simulation will be structured.

The test is made up of true-false type questions. Each question will have two possible solutions. Since John is not going to read the question, both solutions will be equally likely. One of the two solutions will be correct and one will be incorrect. John will have a 50% chance of being correct on each solution.

Since the test has ten questions, the coins will be tossed to represent the outcome of the ten questions on the test. To find the mark associated with this first toss, count the number of coins with the heads facing up and multiply by 10%.

If you would like to see how a random head-tail table can be used to perform this same task, turn to **Extra Help** at the end of this topic.

The results of this part of the simulation is then recorded on the frequency distribution table. If this first toss had a mark of 40%, a tally mark would be placed beside 40% on the table.

This procedure will be repeated 90 times in order to collect reliable data.

To complete the frequency distribution table, the tally marks will be added and written down as frequency for each row.

The results of John's simulation is given in the following frequency distribution table.

Percentage on test	Tally	Frequency
0%		0
10%		2
20%		4
30%		8
40%		18
50%		29
60%		14
70%		11
80%		3
90%		1
100%		0
	Total	90

All that is left to do is to put the information into the experimental probability formula. (Remember this is an experiment.)

$$\text{Experimental probability} = \frac{\text{total number of desirable results}}{\text{total number of results}}$$

$$P(\geq 60\%) = \frac{29}{90} \text{ or } 32\%$$

According to this experiment, John would have a 32% chance of passing the test with a mark of 60% or higher.

Since you already know the probability of a coin toss, you can also find the theoretical probability of this situation. For a theoretical situation, you would get the following frequency distribution table.

Percentage on test	Frequency
0%	1
10%	10
20%	45
30%	120
40%	210
50%	252
60%	210
70%	120
80%	45
90%	10
100%	1
Total	1024

Now you need to count the number of desirable results (you already know the total number of results, 90). You want to count the number of cases that had 60% or more.

Percentage on test	Frequency
60%	14
70%	11
80%	3
90%	1
100%	0
Total	29

To see how this frequency distribution table was found, turn to the Extensions, Topic 1.

Count the number of favourable outcomes.

60%	210
70%	120
80%	45
90%	10
100%	1
Total	386

You can fill this information into the theoretical probability formula.

$$\text{Theoretical probability} = \frac{\text{total number of favourable outcomes}}{\text{total number of outcomes}}$$

$$\begin{aligned}P(\geq 60\%) &= \frac{386}{1024} \\&= \frac{193}{512} \text{ or } 37\%\end{aligned}$$

The theoretical probability of John getting 60% or more on the test is 37%.

No matter which way the probability is calculated, the answer is still the same. Chances are John will not get 60% or more on the test.

There are other simulations which John could use to represent the right-wrong response. He could have used a die and let the even numbers be a right response and the odd numbers be a wrong response. He could have drawn numbers from a hat and done a similar type of correlation. Another method he could use is the selection of numbers from a random number table. Can you think of some others?

In doing this simulation we went through eight steps.

1. The problem was clearly stated and all the necessary information was given.
In this case, John wanted to know what was the probability that he would get 60% or more on a true-false test with 10 questions if he guessed at the answers.

2. The key components were defined.
The key components in this case were the ten questions. You had to simulate whether the solutions given by John for each question were right or wrong.
3. State the underlying assumptions.
You assumed that John was equally likely to pick true or false on all of the questions. This means that he had a 50% chance of being correct on every question. You also assumed that the responses given on any question did not affect the selection in the next question.
4. Select a simulation model that will generate the outcomes for the key component.
You selected the coin toss method since both sides of the coin are equally likely to come up. You let heads represent a correct solution on the test and tails represent an incorrect solution on the test.
5. Define and conduct the first trial.
The trial was defined as being the solution to ten tosses of a coin or one toss of ten coins. The result from each coin represented the solution to one question.

6. Record the findings from the trial.
You made a frequency distribution table to collect the data. Since you were only interested in what percentage of the coins came up heads, you only kept track of that item.

7. Steps 5 and 6 are repeated a large number of times. They should be repeated at least 50 times.
You repeated the experiment 90 times.

8. The information is then summarized and conclusions are drawn.
At last you counted the total number of outcomes where 60% or more was achieved and divided by the total number of outcomes to get the experimental probability of the situation. Then you drew the conclusion that John was not likely to get 60% or more on the exam.

Take a look at another case.

In Canada, 45% of the people have type O blood. These people are called universal donors since their blood can be used for transfusions for people of any blood type. How many people should a blood bank see on average, if they want to get 6 type O blood donors? Assume that the donors arrive independently and randomly at the blood bank.

You want to know how many blood donors you have to see to get 6 type O blood donors. The probability of each blood donor being type O is 45%.

To simulate 45%, you will use a random number table. You will take two digits at a time. The digits 00 to 44 will be used to represent a type O blood donor and the digits 45 to 99 will represent a non-type O blood donor.

Two digit numbers are selected from the random number table until 6 type O blood donors are reached. This is done 50 times. The following is the number of donors that arrived including the sixth type O blood donor.

7. Number of Blood Donors Who Arrived Including the Sixth Blood Donor

Donor	Number of Blood Donors Who Arrived Including the Sixth Blood Donor
14	16
16	18
11	6
14	16
16	14
11	10
10	9
19	16
16	14
11	8
11	10
19	10
15	8
23	11
12	10
10	10
10	13
13	16
11	15
21	11
9	9
15	6

If you take the average number of donors that were seen, you will have a number that will represent the number of donors that will need to be seen to get 6 type O blood donors.

$$\bar{x} = \frac{14 + 16 + 18 + \dots + 10}{50}$$

$$= \frac{644}{50}$$

$$= 12.88 \text{ or } 13$$

\bar{x} is the average or mean.

The blood bank would need to see 13 people.

Using this data, answer the following questions.

- What was the minimum number of donors the bank had to see?
- What was the maximum number of donors the bank had to see?
- Which number occurred most often?

The range of the data went from a possible low value of 6 up to a value of 23. The value 10 appeared nine times in the data.

Now try some questions.

Answer any five of the following questions.

Any tables you may need are provided in **Appendix B**.

1. A hockey player scores an average of 1 out of every 6 shots.

Suppose he makes 8 shots in a game, use a simulation to find the experimental probability that he will achieve the following:

- a. score on all of his shots
- b. score 2 or more goals
- c. score exactly 2 goals
- d. score no goals

(Hint: Use a simulation of the rolling of a die. Let the one of the number represent the situation where the player scores.)

2. Gina runs a small bus between two towns. The bus holds 9 passengers. She found that on average for every ten tickets she sells, one person will not show up for the trip. If Gina sells ten tickets for each trip, use a simulation to find the experimental probability that each of the following will occur.

(Hint: Use the random number table to make your simulation. Let one of the numbers represent the case where one ticket holder will not show up.)

- a. Ten people will show up, but that one will not be able to go on the trip.
- b. There will be one empty seat on the bus.

3. Suppose that the probability of successfully drilling an oil well is 20%.

- a. If a company drills 8 wells, use a simulation to find the experimental probability that each of the following will happen.

(Hint: Use the random number table to make your simulation. Let two of the numbers represent the case where one oil well is successfully drilled.)

- i. One successful oil well has been drilled.
- ii. At least one successful oil well has been drilled.
- iii. Exactly two successful oil wells have been drilled.

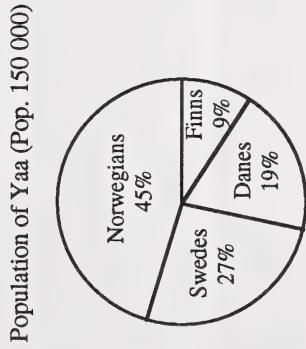
- b. How many wells must be drilled until one oil successful well is found.

4. A car approaching a highway has to come to a complete stop. After stopping, there is a one-third chance that the car will have to wait for a vehicle coming from the left and one-half chance that the car will have to wait for a vehicle coming from the right. Use a simulation to answer the following questions.

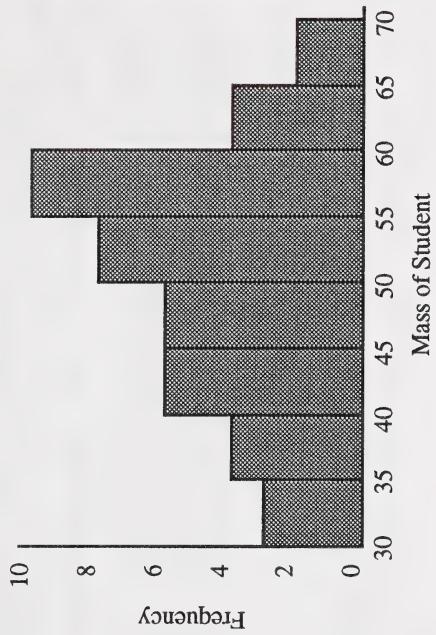
(Hint: Use two differently coloured dice to simulate the approaching vehicles.)

- a. What is the probability that the car can make a left hand turn without delay?
- b. How long should the car expect to wait to make a left hand turn?
- c. How long should the car expect to wait to make a right hand turn?

5. A certain city called Yaa in northern Europe is populated with Scandinavian people. The distribution of the population is depicted in the following graph.



6. The following histogram represents the different masses of students in Ms. Raska's classroom.



Use the information from the histogram to answer the following questions.

- What is the size of the Norwegian population?
- What is the probability that a person is Danish?
- What is the probability that a person is Swedish or Finnish?
- What is the probability that a person is German?
- How many students have a mass less than 45 kg?
- How many students are in Ms. Raska's classroom?
- What is the probability of a student's mass being between 45 and 60 kg?
- What is the probability of a student's mass being higher than 55 kg or less than 35 kg?



For solutions to Activity 4, turn to Appendix A, Topic 1.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help



In your simulation in Activity 4, you wanted to take ten coin tosses at a time. To simulate this, you will take a row of ten characters. (You could take a column of ten characters that were diagonal to each other.) Each of the ten characters will represent getting either a correct answer on the test or an incorrect answer on the test. (H represents a correct answer and T represents an incorrect answer.)

Using a Random Head-Tail Table

The following is a small section of a random head-tail table.

TTHHT	HHTHH	TTHTT	HHHHT	HHHHT	TTTTT	HTHHH	HTHTT	HTHTT	HTHTT
HHTHT	HHTHH	HTHTT	HHHTT	TTHHH	TTHTT	HHHHT	HHHHT	HTHTT	HTHTT
HHTTH	HTHTT	TTHTT	THHHT	HHHHH	HHHHT	HTHTT	THHHT	THHHT	THHHT
HHTTT	HTHTT	HTHTT	THHHT	THHHT	TTHTT	TTHTT	THHHT	THHHT	THHHT
TTHTT	HHHHT	HHHHT	THHHT	THHHT	TTHTT	TTHTT	TTHTT	TTHTT	TTHTT
TTHTT	HTHTT	HTHTT	HHHHT	HHHHT	TTHTT	TTHTT	TTHTT	TTHTT	TTHTT
TTHTT	HTHTT	HTHTT	HHHHT	HHHHT	TTHTT	TTHTT	TTHTT	TTHTT	TTHTT
HHHTT									
HHHTT									
HHHTT									

One of the first things that you will notice about the table is that there are only two different characters present in the table. These two characters represent getting a head or a tail on a coin. The H represents a head and the T represents a tail. Following from the line, TTHTT represents the situation where the following were tossed; tail, tail, head, tail, and a tail.

For this simulation you will use the minimum number of cases, 90. To make sure that you are receiving a random selection, you will select the 90 rows at random from all parts of the table.

Some of the selections are shown in the following section of the table.

HTHTT	HTHTH	HTHHH	HTHTT	HTHHH	TTHTT	HTHTH	HTHTH	HTHTH	HTHTH
HTHTT	TTHTT	THHTT	THHTT	THHTT	HTHTT	HHHHT	HHHHT	HTHTH	THHTT
HHTTH	HTHTT	HTHTT	THHTT	THHTT	THHTT	HTHTT	HTHTT	HTHTT	HTHTT
HHTTH	HHHHT	HHHHT	THHTT						
TTHTT	HHHHT	HHHHT	THHTT						
TTHTT	HTHTT	HTHTT	HHHHT	HHHHT	THHTT	THHTT	THHTT	THHTT	THHTT
TTHTT	HTHTT	HTHTT	HHHHT	HHHHT	THHTT	THHTT	THHTT	THHTT	THHTT
TTHTT	HHHHT	HHHHT	THHTT						
HHHTT									
HHHTT									
HHHTT									

You will consider THHTT HHHTT to be your first row. This row has 4 heads. Therefore, this row represents the case where John would get 4 of the answers correct and would receive 40% on the test.

You will consider T THHHH TTH to be the second row. This row has 5 heads. Therefore, this row represents the case where John would get 5 of the answers correct and would receive 50% on the test.

You will consider HH HHHHT HTH to be the third row. This row has 7 heads. Therefore, this row represents the case where John would get 7 of the answers correct and would receive 70% on the test.

This procedure is continued until all of the 90 cases have been completed.

Now try the following questions.

1. What do the following lines represent?

- a. THTTH
- b. HHHHTH
- c. TTTTH
- d. HHHHH

2. What percentage mark do each of the following lines represent?

- a. TTHHT THHHHH
- b. HTTHHH THTHTH
- c. TTTTT HHHHH
- d. HHHTH HTHTH
- e. TTTTT TTHTT
- f. THHHT HHHHH



For solutions to Extra Help, turn to Appendix A,
Topic 1.

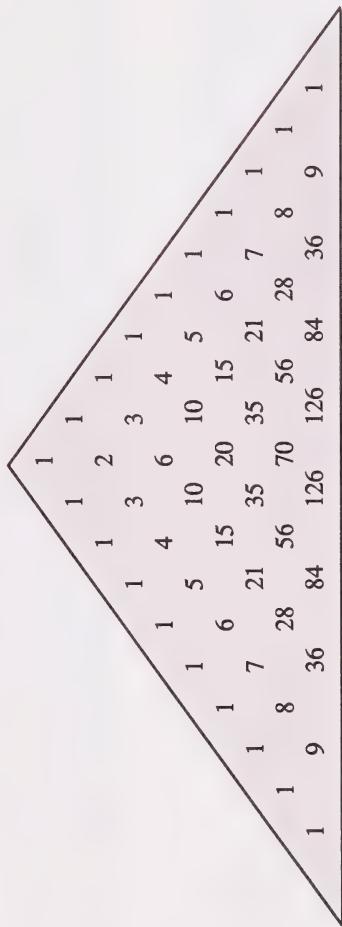


Extensions

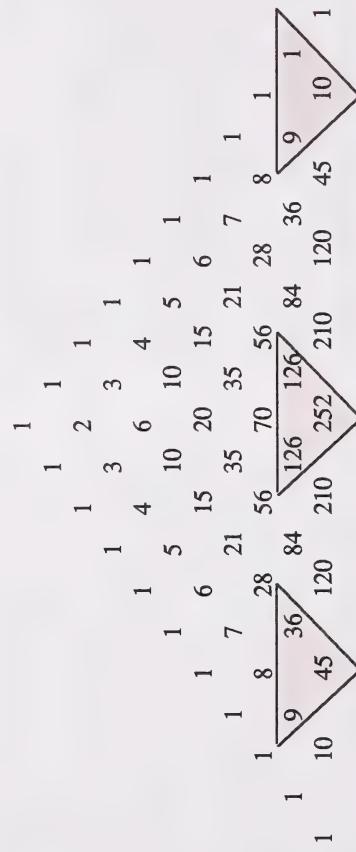
Pascal's Triangle

Pascal's Triangle (or Pascal's Rule) can be used to find the number of outcomes for two option situations such as right-wrong, heads-tails, or 0 - 1.

Take a look at some of the characteristics of the triangle and how you can construct the triangle. Pascal's Triangle has the shape of a triangle (which is why it is called a triangle). To help point this out, a triangle has been placed around the number to emphasize the shape. The following triangle has only ten lines shown and it is important to note that there is an infinite number of lines to Pascal's Triangle.



Each additional row can be found by using the previous row. First, you place the number 1 on each side of the triangle. Next, find the location directly in between and below two of the numbers in the previous row. Add these two numbers and write it down in this location. A small inverted triangle is used in the triangle to show how two numbers are added to give another number in the next row.



But how is the triangle used in finding the theoretical probability?

As was noted at the beginning of Extensions, Pascal's Triangle can be used to find the number of two option situations. To see how this works, take a look at the case where you are tossing a coin.

Case 1: Tossing one coin once.

This case has two outcomes that are equally likely. The second line or what is considered the first row in the triangle represents this case.

1	1
---	---

If you let the heads be the left side of the diagram and the tails the right side, then the number of outcomes would be 1 head and 1 tail. The total number of possible outcomes can be found by either adding up all of the numbers in the row or by raising the number 2 to the exponent that represents the row number. In this case it is the first row, so the number of cases will be 2^1 which is 2.

Case 2: Tossing one coin twice.

This case has four outcomes that are equally likely. They are head-head, head-tail, tail-head, and tail-tail. Since you cannot distinguish between head-tail and tail-head, you treat them as the same case; 1 head, 1 tail. This now gives you three different cases, but they are no longer equally likely. They are shown in the following table.

Outcome	Frequency
2 heads	1
1 head, 1 tail	2
2 tails	1

This case is shown by the second row in the triangle.

1	1	1
---	---	---

This trend continues on infinitely.

Notice that the frequency is identical to the second row. Once again, the heads are on the left side of the triangle.

The total number of outcomes for this case is 2^2 or 4.

Case 3: Tossing one coin three times.

This case has eight outcomes, head-head-head, head-head-tail, head-tail-head, tail-head-head, head-tail-tail, tail-head-tail, tail-tail-head, and tail-tail-tail.

Once again, you give them the following group that is shown in the following frequency table.

Outcome	Frequency
3 heads	1
2 heads, 1 tail	3
1 head, 2 tails	3
3 tails	1

This case is shown by the third row in the triangle.

1	1	1
1	3	3

The frequency is identical to the third row. The total number of outcomes is 2^3 or 8(or $1+3+3+1$).

1	1	1
1	2	1

Looking back at John's test, it had 10 questions. To find the number of possible outcomes for the number of correct answers, look at the tenth row of the triangle. The first number on the left side will represent ten correct solutions, the second number from the left will represent nine correct solutions, and so on.

1	10	45	120	210	252	210	120	45	10	1
10	9	8	7	6	5	4	3	2	1	0
correct										
answers										

There are a total of 2^{10} or 1024 possible outcomes.

Try the following questions.

Any tables you may need are provided in **Appendix B**.

- What is the frequency for each of the different outcomes if one coin was tossed each of the following times?
 - 4 times
 - 6 times
 - 8 times
- What is the total number of outcomes if one coin was tossed each of the following times?
 - 4 times
 - 6 times
 - 8 times
 - 16 times
- What are the 11th and 12th lines of Pascal's triangle?



For solutions to Extensions, turn to **Appendix A, Topic 1**.

Unit Summary

What You Have Learned



In this unit a number of new terms were encountered and used accordingly.

- fair or equal chance
- experimental probability
- theoretical probability
- sample space
- outcomes
- event
- impossible event
- certain event
- compound event
- independent events
- multiplication law
- dependent events
- conditional probability
- mutually exclusive events
- addition law
- simulations

You are now ready to

complete the **Unit Assignment**.

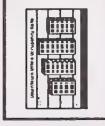
Appendices

✓ Appendix A Solutions

Review

Topic 1 Probability

Appendix B Tables



Random Head-Tail Table

Random Dice Table

Random Number Table

Appendix A Solutions



Review



3. $\frac{720}{1250} = \frac{720}{1250} \times 100\%$
 $= 0.576 \times 100\%$
 $= 57.6\%$

57.6% of the school are girls.

4. 38% of 2000 people
 $38\% \times 2000 = 0.38 \times 2000$
 $= 760$

760 people selected Crost dentifrice.

1. a. $\frac{2}{5} = 0.4$ b. $\frac{3}{4} = 0.75$ c. $\frac{2}{3} = 0.\overline{6}$

d. $\frac{17}{23} = 0.74$ (rounded to 2 significant figures)

e. $\frac{287}{298} = 0.963$ (rounded to 3 significant figures)

2. a. $\frac{34}{100} = \frac{34}{100} \times 100\%$
 $= 0.34 \times 100\%$
 $= 34\%$

b. $\frac{12}{17} = \frac{12}{17} \times 100\%$
 $= 0.71 \times 100\%$
 $= 71\%$

c. $\frac{9}{20} = \frac{9}{20} \times 100\%$
 $= 0.045 \times 100\%$
 $= 4.5\%$

d. $\frac{19}{27} = \frac{19}{27} \times 100\%$
 $= 0.70 \times 100\%$
 $= 70\%$

e. $\frac{123}{234} = \frac{123}{234} \times 100\%$
 $= 0.526 \times 100\%$
 $= 52.6\%$

6. Since the largest group prefer blue cars, the group will probably select a blue car for the lottery.

Exploring Topic 1



Activity 1

4. a. Your answers should be near to those below.

Face of the Coin	Heads	Tails
Number of Times Facing Up	50	50

Determine the probability of an event that is a number between 0 and 1 which describes the likelihood of the occurrence of that event.

1. a. $P(4) = \frac{1}{6}$

b. $P(2) = \frac{1}{6}$

c. You should expect 50 000 heads (one half of 100 000).

2. a. $P(H) = \frac{1}{2}$

b. $P(T) = \frac{1}{2}$

5. Experimental Probability - A fraction between 0 and 1, inclusively, that states the likelihood of an event happening by dividing the number desirable results by the number of results as collected through experiment.

3. a. $P(\text{ace spades}) = \frac{1}{52}$

b. $P(\text{ace}) = \frac{4}{52}$

Theoretical Probability - A fraction between 0 and 1, inclusively, that states the likelihood of an even happening by dividing the number of favourable outcomes by the total number of outcomes when all of the outcomes are equally likely to happen.

c. $P(\text{spade}) = \frac{13}{52}$

d. $P(\text{black}) = \frac{26}{52}$

Theoretical Probability - A fraction between 0 and 1, inclusively, that states the likelihood of an even happening by dividing the number of favourable outcomes by the total number of outcomes when all of the outcomes are equally likely to happen.

b. i. $P(H) = \frac{50}{100}$

$$= \frac{1}{2}$$

ii. $P(H) = \frac{50}{100}$

$$= \frac{1}{2}$$

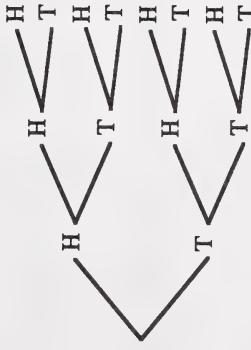
Activity 2

c. Answers will vary.

Find the probability of two or more events occurring together by the application of the multiplication law for independent and dependent events.

1. A compound probability is when one or more events are considered or treated as one event and a conditional probability is when one event must happen before the second event can happen.

2. a.



3. a. $P(b4 \text{ and } g5) = P(b4) \times P(g5)$

$$= \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

b. $P(b2 \text{ and } g\text{-even}) = P(b2) \times P(g\text{-even})$

$$= \frac{1}{6} \times \frac{3}{6}$$

$$= \frac{3}{36}$$

$$= \frac{1}{12}$$

c. $P(b1, b5, \text{or } b6 \text{ and } g\text{-prime}) = P(b1, b5, \text{or } b6) \times P(g\text{-prime})$

$$= \frac{3}{6} \times \frac{3}{6}$$

$$= \frac{9}{36}$$

$$= \frac{1}{4}$$

b. i. There is only one way to get three heads. Therefore,

$$P(3H) = \frac{1}{8}$$

ii. There are three ways to get 2 heads and 1 tail.

$$\text{Therefore, } P(2H, 1T) = \frac{3}{8}$$

b. Answers will vary.

4. a. i. $P(6) = \frac{5}{36}$

ii. $P(8) = \frac{5}{36}$

iii. $P(3, 5, \text{ or } 11) = \frac{8}{36}$

$$= \frac{2}{9}$$

5. a. i. $P(b) = \frac{6}{27}$
 $= \frac{2}{9}$

$P(\text{getting fourth number}) = \frac{3}{46}$

ii. $P(\text{yellow and blue}) = P(\text{yellow}) \times P(\text{blue / yellow})$

$$\begin{aligned} &= \frac{5}{27} \times \frac{6}{26} \\ &= \frac{10}{234} \end{aligned}$$

iii. $P(\text{red and red}) = P(\text{red}) \times P(\text{red / red})$

$$\begin{aligned} &= \frac{9}{27} \times \frac{8}{26} \\ &= \frac{4}{39} \end{aligned}$$

b. Answers will vary.

6. $P(\text{getting first number}) = \frac{6}{49}$ There are 6 numbers out of the 49 numbers that can be your first selection.

$P(\text{getting second number}) = \frac{5}{48}$ There are 5 numbers out of the 48 remaining numbers that can be your second selection.

$P(\text{getting third number}) = \frac{4}{47}$

The probability of winning is 7.15×10^{-8} .

Activity 3

d. $P(\text{number or king}) = P(\text{number}) + P(\text{king})$

$$\begin{aligned} &= \frac{36}{52} + \frac{4}{52} \\ &= \frac{40}{52} \\ &= \frac{10}{13} \end{aligned}$$

Find the probability of the occurrence of one or the other of two events by the application of the addition law.

1. a.

$$\begin{aligned} P(\text{face card or black card}) &= P(\text{face card}) + P(\text{black card}) - P(\text{face and black}) \\ &= \frac{12}{52} + \frac{26}{52} - \frac{6}{52} \\ &= \frac{32}{52} \\ &= \frac{8}{13} \end{aligned}$$

b.

$$\begin{aligned} P(\text{mult. of 3 or heart}) &= P(\text{mult. of 3}) + P(\text{heart}) - P(\text{mult. of 3 and heart}) \\ &= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} \\ &= \frac{22}{52} \\ &= \frac{11}{26} \end{aligned}$$

c. $P(\text{red or black}) = P(\text{red}) + P(\text{black})$

$$\begin{aligned} &= \frac{26}{52} + \frac{26}{52} \\ &= \frac{52}{52} \\ &= 1 \end{aligned}$$

d. $P(\text{red or even}) = P(\text{red}) + P(\text{even}) - P(\text{red and even})$

$$\begin{aligned} &= \frac{18}{38} + \frac{18}{38} - \frac{8}{38} \\ &= \frac{28}{38} = \frac{14}{19} \end{aligned}$$

(Zero is not considered an even number.)

c. $P(\text{2nd twelve or odd}) = P(\text{2nd twelve}) + P(\text{odd}) - P(\text{2nd twelve and odd})$

$$\begin{aligned} &= \frac{12}{38} + \frac{18}{38} - \frac{6}{38} \\ &= \frac{24}{38} \\ &= \frac{12}{19} \end{aligned}$$

d. $P(\text{1st col or even}) = P(\text{1st col}) + P(\text{even}) - P(\text{1st col and even})$

$$\begin{aligned} &= \frac{12}{38} + \frac{18}{38} - \frac{6}{38} \\ &= \frac{24}{38} \\ &= \frac{12}{19} \end{aligned}$$

e. $P(\text{1st twelve or } > 30) = P(\text{1st column}) + P(> 30) - P(\text{1st column and } > 30)$

$$\begin{aligned} &= \frac{12}{38} + \frac{6}{38} - \frac{2}{38} \\ &= \frac{16}{38} \\ &= \frac{8}{19} \end{aligned}$$

3. a. i. $P(2H \text{ or } 3H) = P(2H) + P(3H)$

$$\begin{aligned} &= \frac{3}{8} + \frac{1}{8} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

If you toss a coin three times, the following possibilities may occur.
(H, H, H), (T, T, T),
(T, H, H), (H, T, H),
(H, H, T), (T, T, H),
(T, H, T), (H, T, T)
There are eight combinations.

ii. $P(2H \text{ or } 0H) = P(2H) + P(0H)$

$$\begin{aligned} &= \frac{3}{8} + \frac{1}{8} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

iii.

$$\begin{aligned} P(2T \text{ or } 2H) &= P(2T) + P(2H) \\ &= \frac{3}{8} + \frac{3}{8} \\ &= \frac{6}{8} \\ &= \frac{3}{4} \end{aligned}$$

4. a. $P(\text{white or blue}) = P(\text{white}) + P(\text{blue})$

$$\begin{aligned} &= \frac{3}{25} + \frac{4}{25} \\ &= \frac{7}{25} \end{aligned}$$

b. Answers will vary.

4. a. $P(\text{white or blue}) = P(\text{white}) + P(\text{blue})$

$$\begin{aligned} &= \frac{3}{25} + \frac{4}{25} \\ &= \frac{7}{25} \end{aligned}$$

b. $P(\text{blue or green}) = P(\text{blue}) + P(\text{green})$

$$\begin{aligned} &= \frac{4}{25} + \frac{6}{25} \\ &= \frac{10}{25} \\ &= \frac{2}{5} \end{aligned}$$

c. $P(\text{blue, red, or orange}) = P(\text{blue}) + P(\text{red}) + P(\text{orange})$

$$\begin{aligned} &= \frac{4}{25} + \frac{5}{25} + \frac{7}{25} \\ &= \frac{16}{25} \end{aligned}$$

d. $P(\text{white, blue, or orange}) = P(\text{white}) + P(\text{blue}) + P(\text{orange})$

$$\begin{aligned} &= \frac{3}{25} + \frac{4}{25} + \frac{7}{25} \\ &= \frac{14}{25} \end{aligned}$$

5. a. $P(2 \text{ or } 5) = P(2) + P(5)$

$$\begin{aligned} &= \frac{1}{36} + \frac{4}{36} \\ &= \frac{5}{36} \end{aligned}$$

b. $P(\text{prime or even}) = P(\text{prime}) + P(\text{even}) - P(\text{prime and even})$

$$\begin{aligned} &= \frac{15}{36} + \frac{18}{36} - \frac{1}{36} \\ &= \frac{32}{36} \\ &= \frac{8}{9} \end{aligned}$$

c. $P(\text{odd or } > 9) = P(\text{odd}) + P(> 9) - P(\text{odd and } > 9)$

$$\begin{aligned} &= \frac{18}{36} + \frac{6}{36} - \frac{2}{36} \\ &= \frac{22}{36} \\ &= \frac{11}{18} \end{aligned}$$

d. $P(\text{composite or } < 2) = P(\text{composite}) + P(< 2)$

$$\begin{aligned} &= \frac{21}{36} + \frac{0}{36} \\ &= \frac{7}{12} \end{aligned}$$

$$\begin{aligned} \text{The probability of getting 8 goals out of 8 shots is } 0\%. \\ \text{The probability of getting 8 goals out of 8 shots is } 0\%. \end{aligned}$$

a. $P(8) = \frac{0}{50}$

$$= 0$$

The probability of getting 8 goals out of 8 shots is 0%.

b. $P(2+) = \frac{18}{50}$

$$= \frac{9}{25} \text{ or } 36\%$$

The probability of getting 2 or more goals is 36%.

c. $P(2) = \frac{12}{50}$

$$= \frac{6}{25} \text{ or } 24\%$$

Design and carry out simulations involving events which have known and unknown probabilities.

Activity 4

1. Your solutions should be similar to the following, not the same.

Number of Scores	Frequency
0	14
1	18
2	12
3	5
4	1
5	0
6	0
7	0
8	0

The probability of getting 2 goals is 24%.

d. $P(0) = \frac{14}{50}$

$$= \frac{7}{25} \text{ or } 28\%$$

The probability of getting no goals is 28%.

2. Your solutions should be similar to the following, not the same.

iii. $P(2) = \frac{15}{50} = \frac{3}{10}$ or 30%

Number of Ticketholders Who Didn't Show Up

Frequency
5
8
15
12
10
0

a. $P(10) = \frac{5}{50}$

$$= \frac{1}{10} \text{ or } 10\%$$

One person will not travel on the bus only 10% of the trips.

b. $P(8) = \frac{15}{50}$ or 30%

There will be one seat empty 30% of the trips.

(Hint: Use the random number table to make your simulation. Let one of the numbers represent the case where one ticketholder will not show up.)

3. a. i. $P(1) = \frac{17}{50}$ or 34%

$$= 3.96 \text{ or } 4$$

The probability that 1 successful oil well was drilled is 34%.

ii. $P(1+) = \frac{41}{50}$ or 82%

The probability of drilling at least one successful oil well is 82%.

They would have to drill 4 holes before finding an oil well.

4. Your solutions should be similar to the following.

Start with two differently coloured dice. On the first die, let a 1 or a 2 represent the case where a vehicle is coming from the left. On the second die, let a 1, 2, or 3 represent the case where a vehicle is coming from the right.

The following chart shows how long a vehicle waited to turn left in the first 50 trials.

How long the vehicle waited to turn left.

6	3	0	4	3	4	4	1	2	2
4	1	11	1	1	4	4	4	0	6
7	10	10	1	19	0	5	19	0	6
1	0	0	4	2	0	0	1	10	1
13	3	1	1	2	3	1	1	3	2

The following chart shows how long a vehicle waited to turn right in the first 50 trials.

How long the vehicle waited to turn right.

4	7	7	2	3	2	0	1	3	3
1	1	3	1	0	4	0	2	0	2
0	1	1	0	2	0	0	8	1	3
0	1	1	0	2	2	1	8	0	2
0	0	0	1	3	4	1	0	0	1

a. $P(\text{no wait left}) = \frac{8}{50}$
 $= \frac{4}{25}$ or 16%

There is a 16% chance that the vehicle will not have to wait to make a left turn.

b. Average wait for left = $\frac{7+4+1+5+\dots+2}{50}$

$$= \frac{241}{50}$$

$$= 4.82 \text{ or } 5$$

The vehicle should expect to wait for 5 vehicles before it could turn left.

c. Average wait for right = $\frac{7+7+2+3+\dots+1}{50}$

$$= \frac{86}{50}$$

$$= 1.72 \text{ or } 2$$

The vehicle should expect to wait for 2 vehicles before it could turn right.

5. a. 45% of 150 000 people
 $0.45 \times 150\ 000 = 67\ 500$

There are 67 500 Norwegians.

b. First find the number of Danish people.
 19% of 150 000 people
 $0.19 \times 150\ 000 = 28\ 500$

$$P(\text{Danish}) = \frac{28\ 500}{150\ 000} = \frac{19}{100}$$

The probability of a person being Danish is $\frac{19}{100}$ or 0.19.

c. First you must find the number of Swedes and Finns.
 $36\% \text{ of } 150\,000 \text{ people } (27\% + 9\% = 36\%)$
 $0.36 \times 150\,000 = 54\,000$

$$\begin{aligned} P(\text{Swede or Finn}) &= \frac{54\,000}{150\,000} \\ &= \frac{9}{25} \end{aligned}$$

The probability that a person is Swedish or Finnish is $\frac{9}{25}$.

d. There are no Germans so the probability of a person being German is 0.

$$6. \quad a. \quad 3 + 4 + 6 = 13$$

There are 13 students with a mass less than 45 kg.

$$b. \quad 3 + 4 + 6 + 8 + 10 + 4 + 2 = 43$$

There are 43 students in Ms. Raska's classroom.

c. First you must find the number of students.

$$6 + 8 + 10 = 24$$

$$P(45 \text{ to } 60) = \frac{24}{43}$$

The probability of a student's mass being between 45 and 60 kg is $\frac{24}{43}$.

d. First you must find the number of students.
 $(10 + 4 + 2) + (3) = 16 + 3$
 $= 19$

$$P(> 55 \text{ or } < 35) = \frac{19}{43}$$

The probability of a student's mass being greater than 55 kg or less than 35 kg is $\frac{19}{43}$.

Extra Help

1. a. tail, head, tail, tail, and head
- b. head, head, head, tail, and head
- c. tail, tail, tail, tail, and head
- d. head, head, head, head, and head

2. a. 5 heads - 50%
- b. 5 heads - 50%
- c. 5 heads - 50%
- d. 6 heads - 60%

- e. 1 head - 10%
- f. 8 heads - 80%

Extensions

1. a.

Outcome	Frequency
4 heads	1
3 heads, 1 tail	4
2 heads, 2 tails	6
1 head, 3 tails	4
4 tails	1

b.

Outcome	Frequency
6 heads	1
5 heads, 1 tail	6
4 heads, 2 tails	15
3 heads, 3 tails	20
2 heads, 4 tails	15
1 head, 5 tails	6
6 tails	1

c.

Outcome	Frequency
8 heads	1
7 heads, 1 tail	8
6 heads, 2 tails	28
5 heads, 3 tails	56
4 heads, 4 tails	70
3 heads, 5 tails	56
2 heads, 6 tails	28
1 head, 7 tails	8
8 tails	1

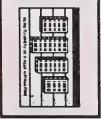
2. a. 2^4 or 16 outcomes

c. 2^8 or 256 outcomes

b. 2^6 or 64 outcomes

d. 2^{16} or 65 536 outcomes

3. 1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1



Appendix B

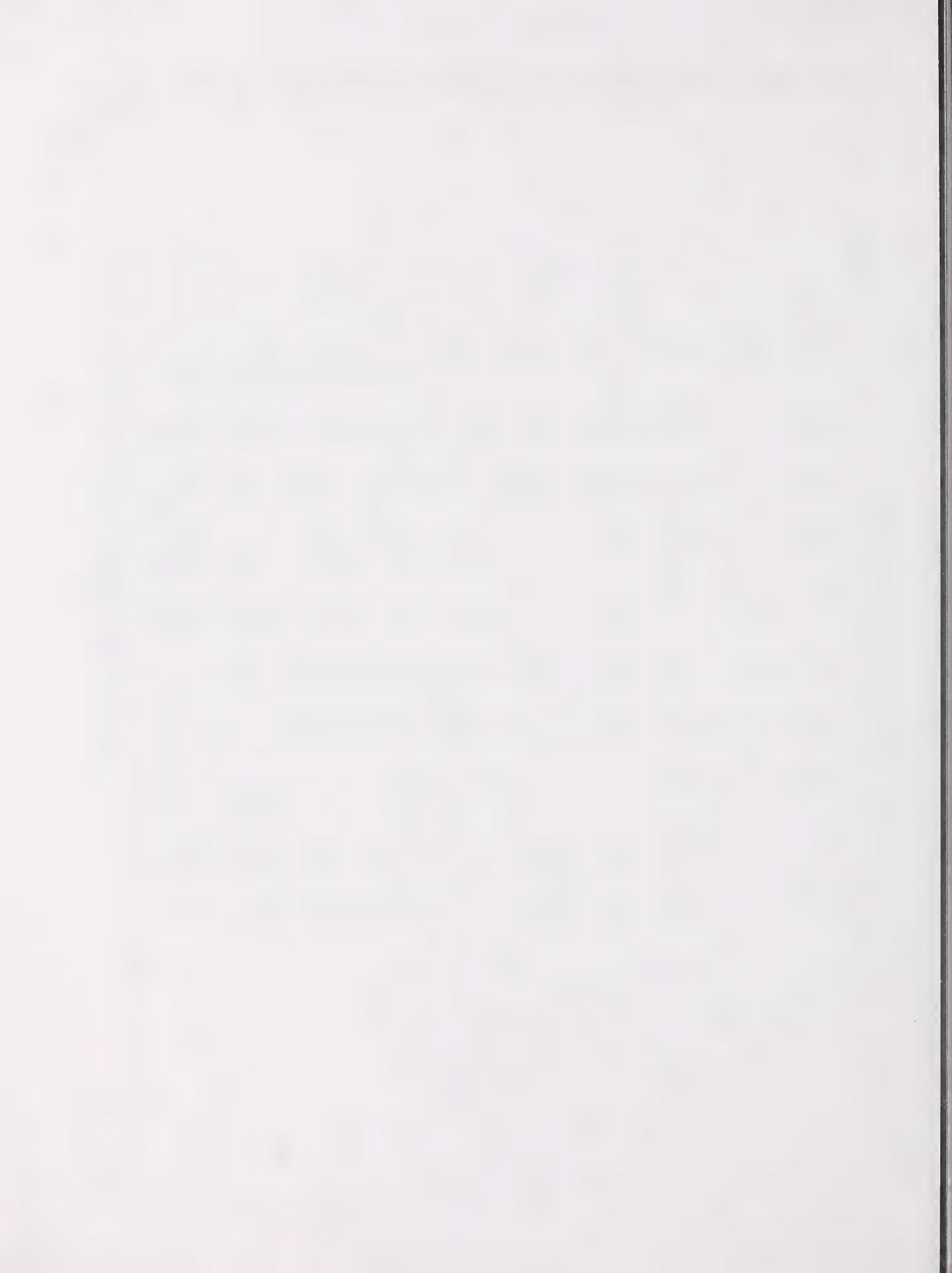
Tables

Random Dice Table

12146	55465	34153	53264	16146	13153	23316
12443	42656	34514	14361	32632	53542	66446
24451	31332	34321	15152	64131	21326	23213
24641	21645	26661	64326	31336	46124	11566
24641	61645	26661	64326	33435	31336	46124
54362	51266	55136	31323	26466	26462	23243
34665	42526	65516	65251	61622	46521	41321
36362	31443	54545	64444	13533	12141	52211
44131	23243	14425	31663	46465	61144	36566
66442	44353	35543	46533	43236	45463	42256
55325	31114	36532	43132	46432	41423	46654
34315	14112	16414	43536	55313	26361	15535
65145	33512	16216	51213	16632	16162	33263
31163	12364	32126	21336	23434	24421	42466
21264	63263	25464	23543	52463	14132	52665
16441	44144	36223	26154	32416	61642	15456
22324	36626	34232	63564	25432	13153	51265
34631	13923	25316	23445	54655	45226	12364
41154	46153	33232	63661	51312	64336	55434
41225	33226	13114	41332	43566	22326	22964
33636	22453	22551	25453	44664	33321	63144
45246	35444	66263	43613	25643	63636	43225
55363	43546	16216	43444	22646	52452	42523
64221	52651	35435	63545	41223	53111	42651
34121	41424	45324	41424	46422	36212	41341
54126	62151	42533	22231	23612	34165	64665
54423	32213	33233	21225	32443	63446	52253
44334	44145	131156	16353	56111	54544	52664
65534	34363	56526	64214	34346	12645	41231
15645	21162	42444	45235	43264	43322	36433
24615	62436	55332	15636	52123	34355	33435
63224	32265	51113	24341	42156	21526	44233
24335	2623	62661	23225	44614	63612	42341
22426	23453	53115	16534	42336	31361	26134
33511	66326	12116	63234	54236	46123	23542
42213	53615	22616	33545	44626	51432	25653
61651	65643	14255	61665	43643	15615	15666
24464	54435	43446	26352	45152	38431	32511
66655	22644	33635	66142	43425	61653	45544
34355	34243	53642	43551	56226	34631	34414
74366	26365	42352	44421	51516	13265	14411
15462	32235	64314	14322	56424	45261	11152
65642	43632	26115	11115	62636	35634	26463
43572	22613	41436	54533	66526	56242	42241
34355	11125	31544	45223	32362	63513	66323
42263	11313	24224	25112	36336	46426	43126
61515	54642	31124	36322	61463	46311	23422
61515	12554	55335	15134	33165	51144	54551
11436	15252	63411	44331	14362	61551	34152
53322	46212	36312	22454	24346	34452	66212
21456	31664	35212	13233	14436	45325	13361

Random Number Table

35707	99631	50314	74845	85466	77160	1396	97957	38623	96977
86512	85588	09766	52143	71897	66545	15729	27928	46277	38631
95059	72986	07270	18649	32027	57654	10000	90855	55843	98089
46407	12448	04852	94168	72872	25581	73139	60361	24619	47450
97623	03970	95838	14590	28791	56532	06504	93647	74485	
12931	37876	29269	19725	89112	51315	67337	66031	97841	87584
76832	60841	25798	28814	83771	79526	05997	87262	65342	71412
43048	04659	13145	70615	32518	45347	45046	21662	40143	15228
05466	43248	19462	30958	64464	13875	98045	48802	88965	38133
36660	19786	24621	29140	80520	71395	64846	46572	02941	94737
36359	66593	94638	66392	68025	91237	20271	07365	23090	53974
16108	45402	37471	67737	65937	22489	83467	01343	01917	42677
35601	11584	53052	70467	45347	38934	21232	88668	39984	48445
31495	10757	70615	32518	67936	62421	57734	36677	76232	88758
93862	19462	30958	65896	18019	22568	83371	27519	61790	22472
72734	08792	40973	35611	20831	54397	13462	64815	72886	50159
22034	05972	54622	10103	77763	56640	12431	92943	32903	83663
91503	54775	51872	75049	18888	43814	75098	68176	80417	61984
01767	40063	87729	35136	77650	03082	07036	76327	12751	84966
36938	95871	71765	40501	40021	16645	20078	03882	21307	84231
40193	88695	28920	21750	06706	71285	96668	87136	73238	06157
47154	07365	23458	82686	09098	93207	98230	36466	53864	91161
87785	11770	85178	59107	23310	76105	73137	91425	26389	03431
13295	19756	48818	77455	03117	01579	25364	69626	93033	12884
41788	88558	08143	05883	37398	27185	45892	31189	34901	26875
92424	64761	72943	45240	06825	57346	06523	12256	53233	47478
98283	79106	77804	35730	16476	98485	98344	91629	31957	48311
36835	02353	44028	93405	76994	54725	46335	86371	97438	
72322	1997	06990	30594	08263	13721	03120	53972	88987	68494
33824	71247	81058	39665	60064	61758	25089	08457	77676	78013
46941	55744	78579	86200	32169	96371	45863	00323	88017	60422
49336	20937	41113	42266	51739	86743	59773	13728	44681	01624
42613	78389	24064	60727	80371	84143	99454	10346	36389	49130
43137	17814	67723	49292	24878	27495	86833	20923	27902	60421
47630	84415	67676	42706	39712	91538	17429	86152	51209	46268
58097	10748	45333	97392	01096	81909	39728	32045	22892	32842
19496	76574	88100	96925	18751	10990	09233	46577	55151	49124
76377	09879	54970	88435	36388	29102	44923	35115	42427	23398
09139	10654	66977	14072	50178	79673	65986	20978	34575	81497
88898	41005	24053	43850	95621	02845	48893	88310	51209	65874
10876	09733	14210	95458	24314	00862	44652	47053	19705	12235
64694	67760	58938	24303	43841	43841	87212	08877	54815	
01110	42704	24131	76252	42281	404438	67852	09537	70113	48284
51433	35043	21918	72430	24797	13563	74129	22257	47929	33868
17964	05742	87740	07619	24903	77440	06017	20795	42251	65850



COURSE SURVEY FOR MATHEMATICS 23

Please evaluate this course and return this survey with your last unit assignment. This is a course designed in a new distance-learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____

Course _____

Address _____

Age under 19

19 to 40

over 40

File No. _____

Date _____

Design

1. This course contains a series of unit booklets and assignment booklets. Do you like the idea of separate booklets?

2. Have you ever enrolled in a correspondence course that arrived as one large volume?

Yes No If yes, which style do you prefer?

3. The unit booklets contained a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

Yes No If yes, explain.

4. Were the questions and directions easy to understand?

Yes No If no, explain.

5. Each topic contains follow-up activities. Which type of follow-up activity did you choose?

- mainly Extra Help
- mainly Extensions
- a variety
- none

Did you find these activities beneficial?

Yes No If no, explain.

6. Did you understand what was expected in the assignment booklets?

Yes No If no, explain.

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

Yes No If no, provide details.

8. Suggestions for audiocassette and videocassette activities may have been included in the course. Did you make use of these media options?

Yes No Comment on the lines below.

Course Content

1. Was enough detailed information provided to help you learn the expected skills and objectives?

Yes No Comment on the lines below.

2. Did you find the work load reasonable?

Yes No If no, explain.

3. Did you have any difficulty with the reading level?

Yes No Please comment.

4. How would you assess your general reading level?

poor reader average reader good reader

5. Was the material presented clearly and with sufficient depth?

Yes No If no, explain.

General

1. What did you like least about the course?

2. What did you like most about the course?

Additional Comments

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

Yes No If yes, approximately how many times? _____

Did you find the staff helpful?

Yes No If no, explain.

2. Were you able to fax any of your assignments?

Yes No If yes, comment on the value of being able to do this.

3. If you were mailing your assignments, how long was it taking for assignment booklets to return?

4. Was the feedback you received from your correspondence teacher helpful?

Yes No Please comment.

Thanks for taking the time to complete this survey. Your feedback is important to us.

Fax Number: 674-6686

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Barrhead, Alberta
T0G 2P0

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